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SUSY, Inflation and the Origin of Matter in the Universe[1]

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Abstract

We consider the Standard Models of particle physics and hot big bang cosmology, and review the theoretical and experimental motivations for extending these models to include supersymmetry and inflation. An obvious extension would be to unite these two models into a single all-encompassing theory. We identify a list of theoretical challenges that such a theory must address, which we illustrate with a simple model - a variant of the next-to-minimal supersymmetric Standard Model - that addresses these challenges.

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1 A Tale of Two Standard Models

The Standard Model (SM) of particle physics provides a description of the fundamental particles and forces present in Nature. It is expressed as a quantum field theory and combines quantum mechanics with special relativity into a single consistent framework. Local *gauge* symmetry is an essential ingredient, and the combined gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ correctly describes electromagnetism and the strong and weak nuclear forces. It can account for the observed low-energy phenomena such as the infinite range of the electromagnetic force and radioactive decay of unstable nuclei in terms of force-mediating quanta. The model has been in place for over 30 years and has been rigorously tested by experiments at high-energy particle accelerators. There are many theoretical reasons to believe in a deeper theory - such as supersymmetry (SUSY) - but it is only in the last few years that new experiments have been able to probe physics beyond the Standard Model (BSM), e.g. massive neutrinos and neutrino oscillations [1], $g_\mu - 2$ measurements [2] and even the recent Higgs candidate at LEP [3].

There is a similar situation in cosmology where the hot big bang (HBB) Standard Model can account for the evolution of the early universe for cosmic times $t \geq t_P \sim 10^{-44}s$ following the big bang, where quantum gravity effects are negligible. The model was developed after the two important discoveries of the cosmic microwave background (CMB) radiation and the Hubble expansion of the universe. Among its many successes, the HBB paradigm can explain nucleosynthesis and reproduce the observed abundances of light elements; and predict a black-body CMB spectrum with the correct temperature of $T_{CMB} \sim 3\,K$. However it has a number of long-standing problems that either require severely fine-tuned initial conditions, or a new theory - such as inflation - that provide observationally-consistent solutions to these problems in a natural way. Recently satellite and balloon-based experiments have yielded evidence that verify the theoretical problems and provide experimental constraints for any extended cosmological model - e.g. COBE density/temperature fluctuations [4], and the BOOMERANG [5], MAXIMA [6] and DASI [7] observations of the angular power spectrum[2].

It would be desirable to combine the two Standard Models (and their extensions) within a single all-encompassing theory - a supersymmetric inflationary model - that provides solutions for the long-standing problems in each SM separately, and is also highly predictive with fewer free parameters [8]. For example, such a theory may eventually unite string theory with cosmology [9] since superstrings offer the best way of unifying all four fundamental forces in a mathematically consistent way.

The layout of the remainder of this review is as follows. In section 2 we discuss how supersymmetry and inflation solve the theoretical and experimental problems of the Standard Models of particle physics and cosmology. In section 3 we introduce the notion of an all-embracing theory that combines supersymmetry and inflation into a single unified framework. Section 3.1 lists the challenges that a supersymmetric inflationary model must address, which we illustrate with a well-studied example in section 3.2 - a variant of the next-to-minimal supersymmetric Standard Model (NMSSM). Section 4 concludes the review.

[2]These observations simultaneously provide information about the matter density and curvature of the universe today.

2 Why go beyond the Standard Models?

In this section we will show how low-energy SUSY and inflation tackle the problems present in the Standard Models of particle physics and cosmology. There are many good references in the literature with further details [10, 11].

2.1 Beyond the Particle Physics Standard Model - Supersymmetry

The Standard Model of particle physics is a quantum field theory that unites the two great successes of twentieth century physics - quantum mechanics and special relativity. It describes the fundamental forces and particles of Nature in terms of the local gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where force-carrying bosons mediate interactions between elementary matter fermions and particles generate masses via their coupling to the Higgs boson. This model has survived rigorous experimental tests at high-energy particle accelerators for over 30 years, but recently experiments have begun to observe hints of new physics that cannot be explained by the Standard Model[1, 2, 3]. These experimental anomalies support the many theoretical reasons - such as the hierarchy and gauge coupling unification problem - which suggest that a new extended model is required.

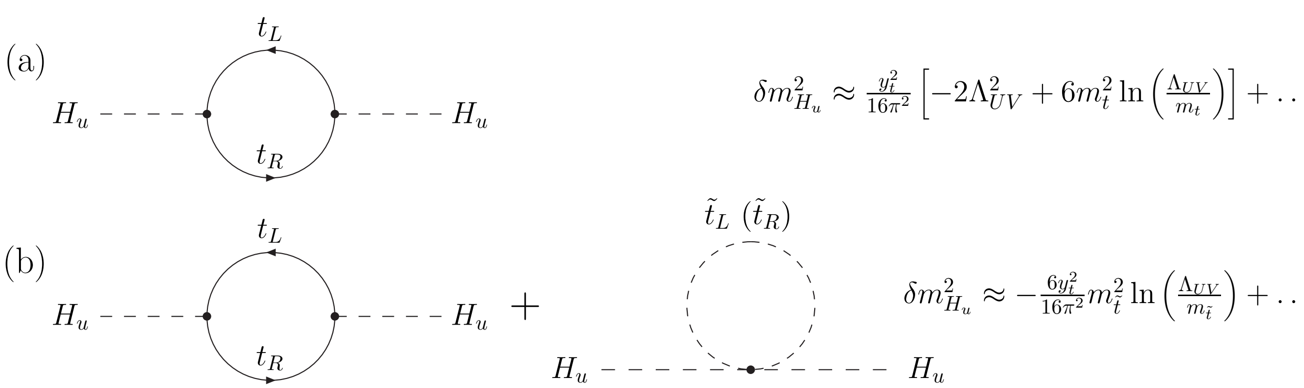

Figure 1: The dominant top (stop) 1-loop corrections to the Higgs mass, where y_t is the top(stop) Yukawa coupling. In the absence of SUSY (a), only top loops contribute, and the radiative correction is found to be quadratically divergent in powers of the ultraviolet momentum cutoff Λ_{UV}. However, when stop loops are included (b), the quadratically divergent pieces cancel out to leave a softer logarithmically divergent correction. In the limit that SUSY is preserved ($m_{\tilde{t}} = m_t$), there is an exact cancellation between the top and stop 1-loop corrections to the Higgs mass.

A leading candidate is supersymmetry - an underlying symmetry that unites fermionic and bosonic degrees of freedom within the same *superfield* multiplets. The minimal extension (MSSM) adds a fermion (boson) superpartner for each boson (fermion) particle in the Standard Model[3]. SUSY combines internal and space-time Poincaré symmetries in a non-trivial way[4]. We know that a theory invariant with respect to these symmetries can provide a realistic model

[3]Notice that the (up-like) Higgs scalar obtains a spin-1/2 Higgsino partner with identical gauge quantum numbers that leads to a gauge anomaly in the theory. This requires that another (down-like) Higgs scalar and Higgsino must be added to cancel this anomaly.

[4]The use of anti-commuting *Grassmann* variables evade the famous Coleman-Mandula No-Go theorem.

of elementary particles and fundamental forces, so it is natural to want to unite internal and space-time symmetries within *global* supersymmetry. Notice that a *gauged* local supersymmetry includes general coordinate transformations and necessarily incorporates a theory of gravity[5].

SUSY solves the gauge hierarchy and naturalness problems by providing a symmetry that protects scalars (Higgs bosons) from acquiring masses of order the underlying gravity (Planck) scale M_P through radiative corrections. Gauge fields are protected by an unbroken gauge invariance and fermions cannot acquire a large mass due to a chiral symmetry. As shown in figure 1, SUSY stabilizes the puzzling ratio: $m_W^2/M_P^2 \simeq 10^{-34}$ by contributing virtual sparticle loops for each particle loop that *soften* the quadratic divergence into a logarithmic divergence. This avoids the unnecessary fine-tuning problems, provided SUSY breaking (and consequently sparticle masses) appear around the TeV scale. Supersymmetry also provides an explanation for the mysterious Higgs mechanism. Electroweak symmetry breaking (EWSB) is triggered by radiative corrections to the Higgs scalar masses, such that 1-loop corrections turn the up-like Higgs scalar squared-mass negative at the origin.

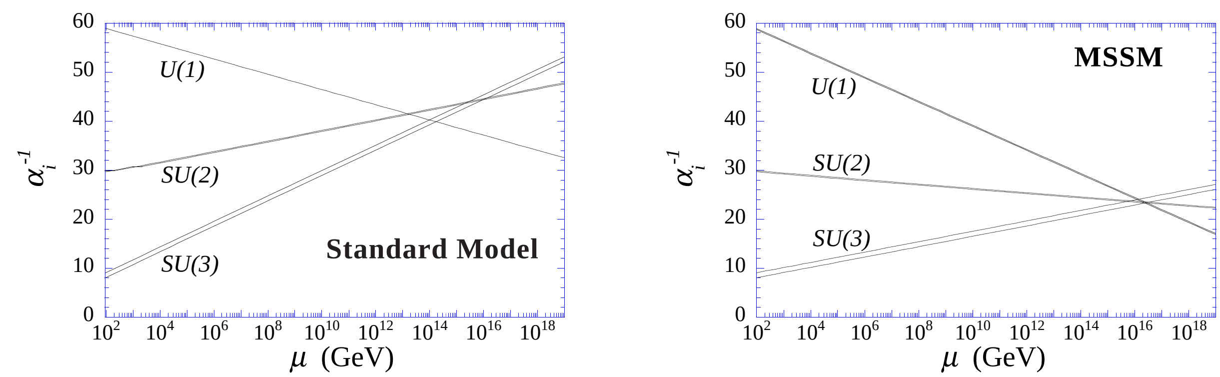

Figure 2: The renormalization group equation (RGE) running of gauge coupling constants α_i as a function of the renormalization scale μ. In the absence of SUSY, the three coupling constants do not meet at a single unified value. However in the MSSM, additional sparticle loops modify the evolution of the gauge couplings so that the coupling constants now meet at a unification scale of $M_X \simeq 10^{16}$ GeV.

SUSY also modifies the gauge coupling renormalization group equation (RGE) running by introducing higher-order loop corrections involving virtual SUSY partners as shown in figure 1. The gauge coupling constants now meet at a scale $M_X \simeq 10^{16}$GeV for $M_{SUSY} \sim 1$ TeV as shown in figure 2. The addition of supersymmetry to grand unified models pushes the potentially dangerous proton decay rate *above* experimental lower bounds. SUSY also offers a solution to the cold dark matter (CDM) problem - the missing mass in the universe - in the form of the lightest supersymmetric particle (LSP) that is very weakly-coupled and stable from decay due to a global R-parity conservation. There are various models predicting the precise nature of the LSP, and neutralinos, gravitinos and axinos have all been considered[6].

However this is not to say that low-energy SUSY is complete. There are still many unan-

[5]Superstrings (so far) provide the only consistent quantum theory involving gravity, and supersymmetry is an essential ingredient.

[6]See L.Roszkowski's plenary talk at this meeting for a discussion.

swered questions, including the precise mechanism responsible for SUSY breaking[7] and the connection of low-energy physics to the proposed underlying superstring theory. However, SUSY is an excellent candidate for TeV-scale physics and its predictions will soon be tested at future accelerators.

2.2 Beyond the Hot Big Bang Model - Inflation

The HBB Standard Model of cosmology combines general relativity and classical thermodynamics to describe the evolution of the universe for cosmic times $t \geq t_P \sim 10^{-44}$ s after the big bang, where quantum gravity effects are negligible. The model can successfully reproduce the observed Hubble expansion of the universe; the existence of the cosmic microwave background radiation with the correct temperature; and can also predict the relative abundances of light elements following nucleosynthesis. In the simplest terms, the HBB model hypothesizes that the universe exploded into existence (perhaps from quantum fluctuations) as a microscopic ball with an unimaginably high temperature. Following unknown quantum effects, the universe contained a hot "soup" of massless particles (quarks, leptons, gauge bosons and Higgs fields) that rapidly cooled as it expanded in size. As it cools, it undergoes a series of phase transitions during which the four fundamental forces separate from a single unified interaction; massless quarks and leptons acquire masses as the Higgs mechanism breaks the electroweak symmetry; and quarks become bound together by the strong force to form hadrons. Eventually the universe cools down sufficiently that nucleosynthesis occurs, where protons and neutrons bind together as nuclei. After the universe cools down further, photons have insufficient energy to prevent electrons from binding to nuclei to form neutral atoms. The photons effectively decouple from matter and no longer interact. This is the epoch when the CMB radiation is formed, and begins to cool down to the temperature we observe today.

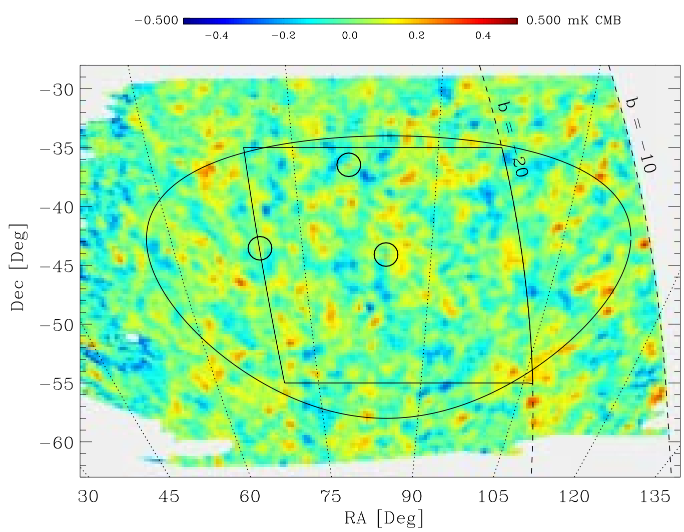

Figure 3: The sky map at 150 GHz, taken from BOOMERANG [5], that shows the temperature anisotopies $\delta T/T \sim 10^{-5}$ in the cosmic microwave background radiation. The location of three quasars are shown as circles.

[7]Many models have been proposed, but it will only be after we have observed supersymmetric sparticle spectra that we will be able to identify the mechanism(s) responsible for SUSY breaking.

Despite these theoretical successes, satellite and balloon-based experiments [5]-[7] have identified features that cannot be explained by the HBB model in a natural way without severe fine-tuning. The BOOMERANG experiment [5] observed a highly uniform CMB temperature in all directions in the sky as shown in figure 3. This level of uniformity requires that all of these regions were causally-connected when photons decoupled from matter, such that all regions equilibrated to a common temperature. Photons can only have travelled a finite distance, at the speed of light, since the CMB radiation was formed. However this horizon is much smaller than the size of the observable universe. So, how did causally-unconnected regions of space achieve a uniform temperature to 1 part in 10^5? A short period of exponential growth - or inflation - prior to the power-law expansion of the HBB model, would solve this "horizon problem" since a small region of causally-connected (and thermalized) space could be instantaneously stretched to a size greater than the observable universe.

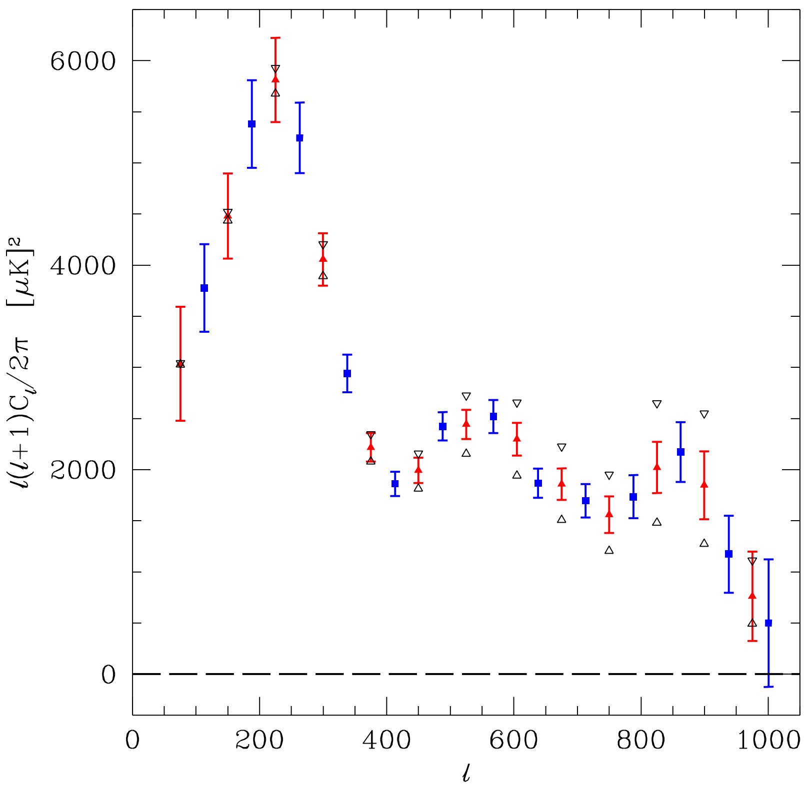

Figure 4: The angular power spectrum of the CMB, as measured at 150 GHz by BOOMERANG and taken from ref. [5]. The blue(square) and red(triangle) points show the results of two independent analyses. The basic result is independent of binning. The vertical error bars show the statistical + sample variance errors on each point. The location of the first peak at $l \approx 200$ is consistent with a flat universe with total density of unity $\Omega_{total} = 1 \pm 0.06$. The presence of the smaller amplitude acoustic peaks effectively rules out models involving topological objects in the early universe such as textures and cosmic strings. Notice that the details of the peaks are dependent on other cosmological quantities such as the Hubble constant and baryon density.

The universe is surprisingly uniform on cosmological scales, but we also know that stars, planets and human-beings exist, which require density fluctuations on smaller scales as shown in figure 3 ($\delta T/T = \delta\rho/\rho \sim 10^{-5}$). How are such density fluctuations generated, while maintaining very large-scale uniformity? Inflation smoothes out any large-scale inhomogeneities in the initial conditions, but regenerates inhomogeneities by stretching quantum fluctuations to an astronomical scale. These fluctuations remain scale-invariant, and lead to the observed large-scale structure in the universe.

Figure 4 shows the angular power spectrum of the CMB as measured at 150 GHz by BOOMERANG [5]. The location of the first peak at a multipole moment $l \approx 200$ corresponds to the angular scale subtended by the Hubble radius at recombination, and is tied to the geometry of the universe[8]. A flat universe has the first peak at $l \sim 200$, and the data provides the best evidence that we live in a flat universe with a total density $\Omega_{total} \approx 1 \pm 0.06$. Resolution of the second and third acoustic peaks in figure 4 provides very strong support for inflation, and effectively rules out models involving topological objects such as textures and cosmic strings. The data also supports a ΛCDM universe in which the energy density is dominated by dark energy (possibly a cosmological constant Λ) and cold dark matter CDM.

3 A Supersymmetric Inflationary Model

In this section we introduce the idea of an all-encompassing theory that combines inflation with supersymmetry, and discuss the motivations for such a model[9]. We also list the issues that such a supersymmetric inflationary model must confront, and we give an explicit example that has already been well studied [12, 13]. Recently, there has been a more detailed discussion of these issues in this supersymmetric inflationary model in ref. [8].

3.1 Challenges for a Supersymmetric Inflationary model

Traditionally, the MSSM derives from a supersymmetric grand unified theory, with an increased unified gauge group such as $SU(5), SU(5) \otimes U(1), SO(10), E_6$ or E_8. This SUSY GUT in turn arises from an effective supergravity model which is the low-energy realization of a superstring theory. The phenomenologically desirable features in the low-energy theory should be derivable (in principle) from the underlying string theory. Unfortunately the lack of knowledge regarding the physical string vacuum state and infinite class of allowed manifolds upon which the theory can be compactified, lead to a confusing ambiguity as to the precise details of the superstring model. However various "bottom-up" approaches to model-building have identified ten important challenges that a supersymmetric inflationary model must be able to address.

1. **μ-term** - the problematic "Higgsino mass" that mixes up and down-like Higgsino fields in the superpotential. Examples of possible solutions include the NMSSM where a gauge-singlet field is added to the MSSM spectrum and generates a μ-term after the singlet acquires a VEV. Alternatively the μ-term may be forbidden in the superpotential by gauge invariance for models with larger gauge groups than the MSSM. Instead it may derive from the Kähler potential through the Giudice-Masiero mechanism [14].

[8] Photon paths diverge (converge) in a negatively (positively) curved universe which leads to a larger (smaller) angular size compared to a flat universe with zero curvature. Negative (positive) curvature pushes the first peak to higher (lower) values of l.

[9] For example, a supersymmetric model of inflation helps to keep the inflaton potential flat.

2. **Strong CP problem** - the non-abelian gauge group $SU(3)_C$ describing the strong interaction allows a CP-violating lagrangian term, where the amount of CP-violation is parametrized by an angle θ. However, experimental tests of the neutron electic-dipole moment show that strong interactions preserve CP-symmetry to a very high accuracy, $|\theta| \leq 10^{-12}$. There is no explanation why θ is so small without fine-tuning. A popular solution imposes an approximate global, axial $U(1)_{PQ}$ Peccei-Quinn symmetry [15] that is broken at a very high energy scale and allows the θ to be rotated away. The breaking of $U(1)_{PQ}$ generates a pseudo-Goldstone boson (axion) that when combined with SUSY offers a cold Dark Matter candidate (axino) [16].

3. **Right-handed neutrinos** - the fermions in the Standard Model are divided into three families of quarks and leptons, where the left-handed fermions transform as doublets and the right-handed fields are singlets with respect to $SU(2)_L$. The absence of right-handed neutrinos in the Standard Model is inconsistent with the observation of (very small) neutrino masses [1]. If we add right-handed neutrinos, we can form a gauge-invariant Yukawa term coupling neutrinos and a Higgs field together that will generate an electroweak-scale Dirac mass after symmetry breaking. The Standard Model gauge symmetry cannot forbid the addition of a right-handed Majorana mass term at a high scale. The see-saw mechanism can now generate heavily suppressed neutrino masses consistent with experiment. Note that grand unified models models based on the extended gauge groups of $SO(10)$ or $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ naturally incorporate right-handed neutrinos since quarks and leptons are unified within the same multiplets.

4. **SUSY breaking** - in the same way that the Higgs mechanism was the final piece of the Standard Model to be discovered, the precise mechanism responsible for SUSY breaking (and the splitting of SM particles and their superpartners) is one of the long-standing problems in supersymmetry. A variety of viable mechanisms have been proposed - such as gravity, gauge, anomaly and gaugino mediation - that make predictions for the supersymmetric mass spectrum. However we will only be able to identify the actual mechanism(s) responsible for breaking supersymmetry following the next generation of accelerators such as LHC and Tevatron Run II.

5. **Inflaton candidate** - inflation is driven by the vacuum energy of a fundamental scalar field - the inflaton - that has so far eluded identification. The NMSSM singlet, and even the two Higgs fields, have been considered as candidates, but none of them provide a sufficiently flat potential. The conventional view is to invoke an additional scalar field (or two such fields in the case of hybrid inflation[17]) and assume that they arise from some deeper theory.

6. **Moduli problems** - big bang nucleosynthesis (BBN) places limits on the time variation of the coupling constants in the SM. In string theory, these couplings are related to the expectation values of moduli fields[10] and can vary in time. Massive moduli fields are produced as non-thermal relics due to vacuum displacement. They could make an embarrassingly large contribution to the critical density of the universe and must be removed [9, 18]. The moduli could decay to other particles, but this would destroy the successes of BBN. Not

[10]Moduli and dilaton fields parametrize the geometry of the theory, especially "flat directions" in field space.

only is moduli over-production a problem, but the dilaton must also be stabilized at a value that does not correspond to weak coupling in string theory[11].

7. **Gravitino problems** - in supergravity models constrained by Big Bang nucleosynthesis[12], the gravitino is predicted to have a mass of $m_{3/2} \sim 10^3\, GeV$ which is comparable to the scale of SUSY breaking in the visible sector. The gravitino has very weak couplings (gravitational in origin) so that it decouples very early in the evolution of the universe, leaving a large relic abundance after nucleosynthesis. These slowly-decaying gravitinos ($\tau_{3/2} \sim 10^3 s > t_{BBN}$) produce a large number of high energy photons that can dilute baryons, and photodissociate nuclei to affect the agreement with the observed abundances. These relic gravitinos need to be removed somehow [20].

8. **Baryogenesis**[13] - the problem of the origin of baryons, specifically the source (and stabilization) of the observed baryon-antibaryon asymmetry. The inclusion of right-handed neutrinos can lead to baryogenesis via leptogenesis [21]. Today baryons contribute $\Omega_b \sim 0.05$ to the energy density of the universe, where recent observations suggest that a total energy density of unity is required. This leaves the problem of the missing mass in the universe that may occur in the form of cold Dark Matter (CDM) or as "Dark Energy".

9. **Cold Dark Matter candidates**[14] - observations require the existence of so-far unobserved and very weakly-interacting particles - cold Dark Matter (CDM) - that contribute $\Omega_{CDM} \sim 0.3$ to the total energy density. Supersymmetric extensions of the SM offer CDM candidate particles - gravitinos, neutalinos, axinos and neutrinos - that are stable and therefore cannot decay into SM matter particles.

10. **Dark Energy problems** - there is still $\Omega_\Lambda \sim 2/3$ of the total energy density in the universe that has not been identified. The so-called "Dark Energy" density can either be time-independent (cosmological constant), or vary with time (quintessence). However we need to understand why $\Omega_\Lambda \sim \Omega_{matter}$ now [22]. Recent work has considered how the dark energy density can be deduced from a supersymmetric model of inflation using only the CMB temperature and Hubble constant as input parameters [8].

3.2 ϕNMSSM and Hybrid Inflation - an explicit example

We will now outline a model that one of us (S.F.K.) has worked on [12] - a variant of the NMSSM [23] - that addresses the challenges set down in section 3.1. There is a summary of the model in section 8.7 of ref. [24]. We closely follow the recent analysis in ref. [8].

This variant of the NMSSM has the following superpotential terms involving the standard Higgs doublets and two gauge singlet fields ϕ (inflaton) and N.

$$W = \lambda N H_u H_d - k\phi N^2 \tag{1}$$

[11] Dilaton stabilization has been recently discussed in the context of type I string theory [19].

[12] SUSY is broken at an intermediate scale $\sim 10^{11}\, GeV$ in a hidden sector and communicated to the visible sector via gravity mediation.

[13] Notice that the next three challenges have no answer in the Standard Model, and strongly depend on the particular inflationary model.

[14] This is the subject of L. Roszkowski's plenary talk at this meeting.

Notice that the standard NMSSM is recovered if we replace the inflaton ϕ by N. However this leads to the familiar domain wall problems arising from the discrete Z_3 symmetry. In this new variant, the Z_3 becomes a global Peccei-Quinn $U(1)_{PQ}$ symmetry [15] that is commonly invoked to solve the strong CP problem. This symmetry is broken in the true vacuum by non-zero ϕ and N VEVS, where the axion is the pseudo-Goldstone boson from the spontaneous symmetry breaking and constrains the size of the VEVS. For the inflation model to work, axion physics require $\langle\phi\rangle \sim \langle N\rangle \sim 10^{10} - 10^{13}$ GeV.

The μ-term of the MSSM is identified as

$$\mu \equiv \lambda\langle N\rangle \sim 10^3\, GeV \tag{2}$$

which implies that $\lambda \sim 10^{-10}$ if $\langle N\rangle \sim 10^{13}$ GeV. A model with such large VEVS gives an intermediate scale solution to the μ-problem, and will have collider signatures as discussed in ref. [25]. The question remains why the coupling constants λ, k appear to be *unnaturally* small in comparison to the larger gauge-singlet VEVS at 10^{13} GeV. This problem has been addressed in ref. [26] where such tiny couplings arise from non-renormalizable operators.

We can make the ϕ-field real by a choice of the (approximately) massless axion field. We will now regard ϕ and N to be the real components of the complex singlets in what follows. When we include soft SUSY breaking mass terms, trilinear terms $A_k k\phi N^2 + h.c.$ (for real A_k) and neglect the $H_u H_d$ superpotential term, we have the following potential[15]:

$$V = V_0 + k^2N^4 + \frac{1}{2}m^2(\phi)N^2 + \frac{1}{2}m_\phi^2\phi^2 \tag{3}$$

$$where \quad m^2(\phi) = m_N^2 + 4k^2\phi^2 - 2kA_k\phi \tag{4}$$

We can identify the various elements of the potential: V_0 arises from some other sector of the theory, SUGRA for example, and dominates the potential; the soft SUSY breaking parameters A_k and m_N are generated through some gravity-mediated mechanism with a generic value of $\mathcal{O}(TeV)$; and m_ϕ comes from no-scale SUGRA, and vanishes at the Planck scale[16].

Note that the N-field is destabilized if ϕ lies between the values:

$$\phi_c^{\pm} = \frac{A_k}{4k}\left(1 \pm \sqrt{1 - \frac{4m_N^2}{A_k^2}}\right) \tag{5}$$

where we are assuming that $4m_N^2 < A_k^2$ in the following analysis.

In order to discuss inflation as illustrated by figure 5, we need to specify the sign of the inflaton mass squared m_ϕ^2. If $m_\phi^2 > 0$ (hybrid inflation) then, for $\phi > \phi_c^+$, N will be driven to a local minimum (false vacuum) with $N = 0$. ϕ will roll towards the origin and m_ϕ^2 will change signs and become negative for $\phi \approx \phi_c^+$. Following this sign change, the potential develops an instability in the $N = 0$ direction and both singlets roll down towards the global minimum (true vacuum) at:

$$\langle\phi\rangle = \frac{A_k}{4k} \tag{6}$$

$$\langle N\rangle = \frac{A_k}{2\sqrt{2}k}\sqrt{1 - \frac{4m_N^2}{A_k^2}} = \sqrt{2}\left|\phi_c^{\pm} - \langle\phi\rangle\right| \tag{7}$$

[15] Notice that since ϕ and N are regarded as the real components of the complex singlets, they must have the same overall factor of 1/2 in their mass terms.

[16] It is generated through radiative corrections such that $m_\phi^2 \sim -k^2A_k^2 \sim -(100eV)^2$.

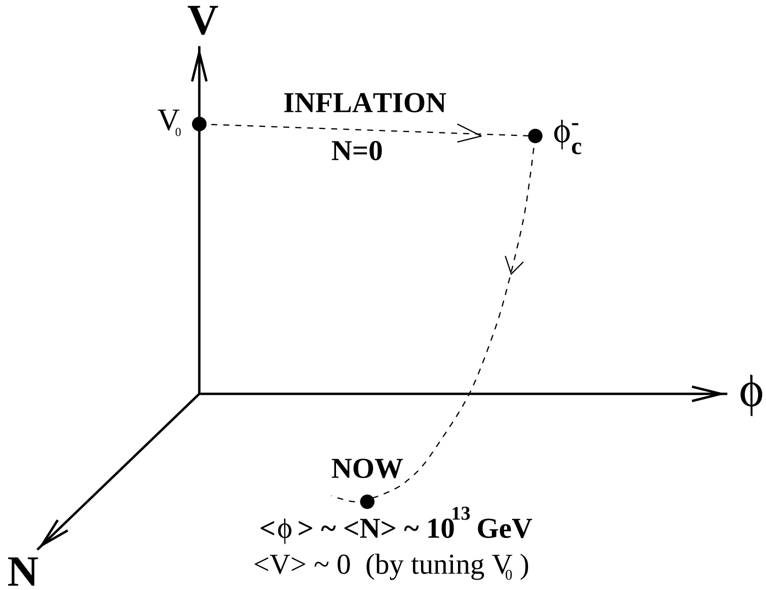

Figure 5: During (inverted hybrid) inflation the singlet N is trapped at the origin and the inflaton rolls towards a critical value at ϕ_c^-, whereupon the potential acquires an instability and rolls down towards the true global minimum (NOW) where the singlets have VEVS $\sim 10^{13}GeV$ and the cosmological constant vanishes in agreement with observation. Notice that ϕ and N are the real components of the complex singlets.

that signals the end of inflation.

However if $m_\phi^2 < 0$ (inverted hybrid inflation), we suppose that during inflation $\phi < \phi_c^-$, and the inflaton rolls away from the origin, eventually reaching ϕ_c^- and ending inflation at the same global mimimum as before. Notice that the global (true vacuum) VEV $\langle\phi\rangle$ lies between ϕ_c^- and ϕ_c^+, so either hybrid or inverted hybrid inflation is possible[17] depending on the sign of the inflaton mass squared m_ϕ^2.

We will also ignore the tiny effect of m_ϕ when we calculated the true vacuum VEVS to obtain the following order of magnitude results:

$$A_k \sim k\phi_c^\pm \sim k\langle N\rangle \sim k\langle\phi\rangle \sim 1TeV \tag{8}$$

For VEVS at the axion scale $\sim 10^{13}GeV$, we require that $k \sim \mathcal{O}(10^{-10})$, and λ must also take a similarly small value since the combination $\lambda\langle N\rangle$ provide the μ-parameter. Notice that the SUGRA-derived potential contribution V_0 exactly cancels with the other terms (by tuning) to provide agreement with the observed small cosmological constant. Thus we assume:

$$V(0) = -V(\langle\phi\rangle, \langle N\rangle) = k^2\langle N\rangle^4 = 4k^2(\phi_c^\pm - \langle\phi\rangle)^4 \tag{9}$$

We may set $N = 0$ during inflation, so the potential of eq.(3) simplifies to:

$$V = V(0) + \frac{1}{2}m_\phi^2\phi^2 \tag{10}$$

[17] However the radiative corrections to the inflaton mass actually give inverted hybrid inflation as shown in figure 5.

During inflation, the inflaton field ϕ is supposed to be on a region of the potential which satisfies the following *flatness conditions*[18]:

$$\epsilon \equiv \frac{1}{2}\tilde{M}_P^2\left(\frac{V'}{V}\right)^2 \ll 1 \tag{11}$$

$$|\eta| \equiv \left|\frac{\tilde{M}_P^2 V''}{V}\right| \ll 1 \tag{12}$$

where $V'(V'')$ are the first (second) derivatives of the potential, and $\tilde{M}_P^2 = M_P^2/8\pi$ is the reduced Planck mass. From eqs.(11,12), the slow roll conditions are given by:

$$\epsilon_{\mathcal{N}} = \frac{M_P^2 m_\phi^4 \phi_{\mathcal{N}}^2}{16\pi V(0)^2} \ll 1 \tag{13}$$

$$|\eta_{\mathcal{N}}| = \frac{M_P^2 |m_\phi^2|}{8\pi V(0)} \ll 1 \tag{14}$$

where $\epsilon_{\mathcal{N}}, \eta_{\mathcal{N}}$ and $\phi_{\mathcal{N}}$ are evaluated around $\mathcal{N} = 60$ e-folds before the end of inflation[19] and $V(0)$ is the dominant term in eq.(10) during inflation, $\phi_{\mathcal{N}} = \phi_c^{\pm} e^{\eta\mathcal{N}} \approx \phi_c^{\pm}$. The height of the potential during inflation is approximately constant and given by:

$$V_0^{1/4} \sim k^{1/2}\langle N\rangle \sim 10^8 GeV \tag{15}$$

We need to check that we can reproduce the correct level of density perturbation - responsible for the large scale structure in the universe - according to the COBE anisotropy measurements, where the spectrum of perturbations is given by[28]:

$$\delta_H^2 = \frac{32V(0)}{75M_P^4\epsilon_{\mathcal{N}}} \tag{16}$$

with the COBE value, $\delta_H = 1.95 \times 10^{-5}$ [29]. Writing $\phi_c^{\pm} \sim \phi_c$ and combining eqs.(8, 11, 15, 16), we obtain the order of magnitude constraint:

$$|km_\phi| \simeq 8\left(\frac{8\pi}{75}\right)^{1/4} \delta_H^{-1/2} \frac{(k\phi_c)^{5/2}}{M_P^{3/2}} \simeq 10^{-18}\, GeV \left(\frac{k\phi_c}{1\, TeV}\right)^{5/2} \tag{17}$$

which is adequate to broadly satisfy the slow-roll conditions of eqs.(11,12)

$$|\eta_{\mathcal{N}}| \simeq \frac{M_P^2}{8\pi}\frac{|km_\phi|^2}{\left(\sqrt{2}k\phi_c\right)^4} \sim 10^{-12}, \tag{18}$$

$$\epsilon_{\mathcal{N}} \sim \frac{M_P^2}{16\pi}\frac{|km_\phi|^4}{\left(\sqrt{2}k\phi_c\right)^8}\phi_{\mathcal{N}}^2 \sim 4\pi\frac{\phi_{\mathcal{N}}^2}{M_P^2}\eta_{\mathcal{N}}^2 \tag{19}$$

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[18] See ref. [27] for further details.

[19] The scale factors $a(t)$ before and after inflation are related by $a(t_{after})/a(t_{end}) = e^{\mathcal{N}}$, where $\mathcal{N}$ is called the number of "e-folds".

This (inverted) hybrid model predicts a very flat spectrum of density perturbations, with no appreciable deviation in the spectral index, $n = 1 + 2\eta - 6\epsilon$ from unity which is consistent with observations and predictions from an $n = 1$ scale-invariant Harrison-Zel'dovich spectrum. Notice that the COBE results require the product $|km_\phi|$ to be extremely small, which implies that the inflaton mass is in the electronvolt range ($m_\phi \sim eV$) when we take $k \sim 10^{-10}$ which is motivated by axion physics as discussed earlier.

Inflation ends with the singlets ϕ, N oscillating about their global minimum. Although the final reheating temperature is estimated to be of order 1 GeV [12], during the reheating process the effective temperature of the universe (as determined by the radiation density) can be viewed as rapidly rising to $V_0^{1/4} = k^{1/2}\langle N\rangle \sim 10^8 GeV$ then slowly falling to the final reheat temperature during the reheating process [13]. This reheating gives entropy to the universe. Non-perturbative effects can produce particles with masses up to the potential height, i.e. $m \leq V_0^{1/4} \sim 10^8 GeV$ (preheating) [30]. We can check that problematic axions and gravitinos are not over produced [20, 31]. The superpotential is modified since Higgses and right-handed sneutrinos $\tilde{\nu}_R$ are copiously produced during this preheating phase via the couplings λ and k to the oscillating inflaton fields.

$$W = \lambda N H_u H_d - k\phi N^2 + Y_\nu L \cdot H_u \nu_R + M\nu_R \nu_R \tag{20}$$

These additional superpotential terms solve the problem of non-zero neutrino masses. The right-handed neutrinos are SM gauge-singlets and so a heavy Majorana mass term can be added at a high energy scale. Neutrino Dirac masses are then generated by electroweak symmetry breaking from the Yukawa coupling terms in the superpotential. The see-saw mechanism generates two mass eigenvalues - one is very large (above the reach of current detection) - and the other is very light and therefore consistent with the recent experimental constraints.

Now consider the origin of matter in the universe[20]. Baryons originate from sleptogenesis [21] via the out-of-equilibrium decay of right-handed sneutrinos ($\tilde{\nu}_R$) and Higgses that violate lepton number (and hence $B - L$) asymmetry before subsequently converting into baryon number asymmetry through sphaleron interactions. From the perspective of inflation, the conventional leptogenesis picture will change if the reheat temperature is below the mass of the lightest right-handed neutrino. Notice that, unlike the usual hot big bang scenario, the out-of-equilibrium condition is automatically satisfied during reheating, and the production mechanism of right-handed neutrinos is totally different and due to direct or indirect couplings to the inflaton field.

In the standard hot big bang scenario, the baryon asymmetry is given by:

$$Y_b \sim \frac{d\epsilon}{g^*} \tag{21}$$

where ϵ is the lepton number asymmetry produced in the decay of the lightest right-handed neutrino

$$\epsilon = \frac{\Gamma\left(\tilde{\nu}_R \to \tilde{l} + H_u\right) - \Gamma\left(\tilde{\nu}_R \to \bar{\tilde{l}} + \overline{H}_u\right)}{\Gamma\left(\tilde{\nu}_R \to \tilde{l} + H_u\right) + \Gamma\left(\tilde{\nu}_R \to \bar{\tilde{l}} + \overline{H}_u\right)} \tag{22}$$

[20]We will only give a summary of the results since a detailed discussion is given in ref. [13].

g^* counts the effective number of degrees of freedom[21], and d is the dilution factor. However for the non-standard leptogenesis picture outlined above, the baryon asymmetry is given by:

$$Y_b \sim \frac{\gamma\epsilon(cV_0)^{1/4}}{M_1} \tag{23}$$

where c is the fraction of the total vacuum energy density converted into right handed neutrinos (mass M_1) due to preheating, and γ accounts for dilution due to entropy production during reheating.

There are two primary contributions to the cold Dark Matter candidate particles, either supersymmetric partners or the Peccei-Quinn axion.

- Neutralino [32] / singlino [25] / inflatino [33] / axino [16] - the Higgs bosons can decay into radiation and a neutralino $H_u, H_d \to \gamma + \tilde{\chi}^0$, where the neutralino can subsequently decay into an inflatino $\tilde{\phi}$, singlino $\tilde{N}$ or axino $\tilde{a} \sim \alpha\tilde{\phi} + \beta\tilde{N}$, provided that they are lighter than the neutralino $\tilde{\chi}^0$.

- Axions - relativistic axions are produced during preheating and so they are red-shifted away; non-relativistic axions are generated at the QCD scale by the misalignment mechanism, and make CDM candidates.

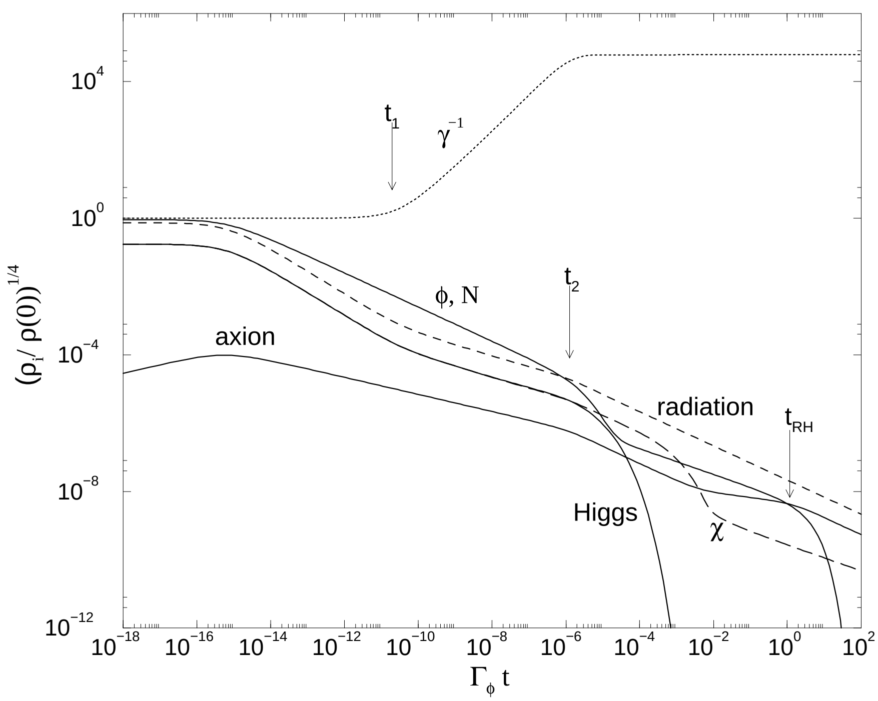

Figure 6: Evolution of the energy densities of the singlets (ϕ, N), axions, Higgses, radiation (dashed line) and neutralinos (long-dashed line). The full analysis and figure are taken from ref. [13].

The time evolution of the various particle densities can be determined by solving a series of coupled Boltzmann equations [13] for a particular choice of parameters as shown in figure 6. In

[21]For the SM, $g^* = 106.75$, and for the MSSM $g^* = 228.75$.

principle, we can calculate the densities of neutralinos, radiation, relativistic axions and baryons at reheating time t_{RH} (defined as the time at which the oscillating singlet energy density rapidly decayed to zero [13]) which represents the start of the hot big bang.

The important point to emphasize for a given model is that, at this time t_{RH}, the Boltzmann equations allow us to calculate the energy densities of matter and radiation (at t_{RH}). As pointed out recently [8], this allows us to calculate the dark energy density (in principle) without having a specific dark energy model in mind, but only inputting the CMB temperature T_{CMB} and Hubble constant.

Finally we will mention a few important points about SUGRA, where the Kähler potential can be split into separate Kähler potential and superpotential terms that are functions of the dilaton (S), an overall moduli (T), inflaton (ϕ) and gauge singlet (N) fields. These functions include non-perturbative terms to stabilize the dilaton and moduli potentials.

$$\begin{aligned} G &= K + \ln |W|^2 && (24) \\ K &= -3\ln(\rho) + \frac{\beta_{np}}{\rho^3} - \ln(S+S^*) + \hat{K}_{np}(S) && (25) \\ W &= -k\phi N^2 + \Lambda^3 e^{-S/b_0} + \dots && (26) \end{aligned}$$

where $\rho = (T+T^*) - \phi^*\phi - N^*N$, and we assume an overall modulus T. Notice that β_{np}/ρ^3, $\hat{K}_{np}(S)$ and $\Lambda^3 e^{-S/b_0}$ arise through non-perturbative mechanisms.

Notice that eq.(26) has a no-scale structure with $m_\phi = m_N = 0$ at tree-level. As mentioned earlier, the cosmological constant can be tuned to zero by an appropriate choice of $V_0 = |F_S|^2 + |F_\rho|^2 \sim (10^8\, GeV)^4$.

During inflation, the dilaton S and "moduli" ρ are stabilized at their respective minima since as the inflaton ϕ rolls (and $N=0$), the overall modulus field T adjusts to keep the combination $\rho = (T+T^*) - \phi^*\phi$ fixed. After inflation, S and ρ only shift by $\sim 10^{-10}$, and so there is no moduli problem. However, it is important to clarify the connection with string theory, e.g. stabilization of the dilaton potential in type I string models [19].

4 Final Thoughts

We will soon see considerable progress in cosmology and supersymmetric particle physics due to the observations of the Map and Planck explorer satellites, and the Tevatron and LHC accelerators. These experiments will accurately measure the fundamental parameters such as the abundances Ω_b, Ω_{CDM} and Ω_Λ present in the universe and the supersymmetric mixing angles and soft parameters. However *ab initio* predictions of these parameters are difficult to obtain, but within the framework of an all-embracing supersymmetric inflationary model there will be fewer variables (and more predictivity) since the same parameters control both inflation *and* collider physics [8].

The NMSSM variant discussed in section 3.2 is an example of such a model which addresses the ten theoretical challenges outlined in section 3.1. However we admit that it is a long way from being able to make accurate predictions for Ω_b, Ω_{CDM} and Ω_Λ. In particular there also needs to be an explanation why $\rho_\Lambda^{1/4} \sim M_W^2/M_P$. However, this model is a step towards more realistic supersymmetric inflationary models that may also incorporate superstring theory.

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